

C.U.SHAH UNIVERSITY

Summer Examination-2019

Subject Name : Mathematical methods -I

Subject Code : 5SC03MAM1

Branch: M.Sc. (Mathematics)

Semester : 3

Date : 13/03/2019

Time : 02:30 To 05:30

Marks : 70

Instructions:

- (1) Use of Programmable calculator and any other electronic instrument is prohibited.
- (2) Instructions written on main answer book are strictly to be obeyed.
- (3) Draw neat diagrams and figures (if necessary) at right places.
- (4) Assume suitable data if needed.

SECTION – I

- Q-1 Attempt the Following questions (07)**
- a. Define : Odd function. (01)
 - b. What is period of $f(x) = \sin 2x + \cos x$? (02)
 - c. Give one example of function which is neither even nor odd. (02)
 - d. Write any one application of Z-transform. (02)

- Q-2 Attempt all questions (14)**
- a. Define : Fourier series of odd function on $(-1, 1)$ (02)
 - b. Find Fourier series of e^{ax} in the interval $(0, 2\pi)$. (06)
 - c. What is half range Fourier cosine series ? Find half range Fourier cosine series of $f(x) = x$ in $(0, 2)$. (06)

OR

- Q-2 Attempt all questions (14)**
- a. State Dirichlet's conditions for the existence of Fourier series (02)
 - b. Find Fourier series of $f(x) = x + x^2$; where $-2 \leq x \leq 2$. (06)
 - c. Find half range fourier sine series of $f(x) = e^x \cos x$ in $(-2, 0)$. (06)

- Q-3 Attempt all questions (14)**
- a. Prove that $F\{f(x)\cos ax\} = \frac{1}{2}\{F(s+a) + F(s-a)\}$ (02)
 - b. Find fourier transform of $e^{-\frac{x^2}{2}}$. (06)
 - c. Find the Fourier sine transform of $f(x) = \begin{cases} 0 & ; 0 < x < a \\ x & ; a < x < b \\ 0 & ; b < x \end{cases}$ (06)

OR

- Q-3 Attempt all questions (14)**



- a. Define : Fourier transform . (02)
- b. Find Fourier transform of $e^{-a|x|}$. (06)
- b. Find the Fourier integral representation of $f(x)=\begin{cases} \sin x & ; 0 \leq x \leq \pi \\ 0 & ; x > \pi \end{cases}$ (06)

SECTION – II

Q-4 Attempt the Following questions (07)

- a. What is Z-transform? (01)
- b. Write any one application of Laplace transform. (02)
- c. Define: Laplace inverse transform. (02)
- d. Find $L(2^t)$. (02)

Q-5 Attempt all questions (14)

- a. Show that $L(\sin at) = \frac{a}{s^2 + a^2}$. (02)
- b. State and prove laplace transform of periodic function. (06)
- c. Evaluate (1) $L(\cos t + \sin t)$ (2) $L(t^2 \sin 2t)$ (06)

OR

Q-5 Attempt all questions (14)

- a. Find $L(\cos 2t + 2t^2)$. (02)
- b. State and prove Laplace transform of n^{th} derivative of function . (06)
- c. Find Laplace transform of $f(t) = \begin{cases} 1 & ; 0 \leq t < 1 \\ t & ; 1 \leq t < 2 \\ 2 & ; 2 \leq t \end{cases}$ (06)

Q-6 Attempt all questions (14)

- a. State first shifting theorem of Laplace transform. (02)
- b. State and prove convolution theorem. (06)
- c. Find inverse Laplace transform of (06)
- (1) $\frac{s+1}{(s^2-1)(s+2)}$ (2) $\frac{s}{(s^2+4a^2)(s-1)}$.

OR

Q-6 Attempt all Questions (14)

- a. Define : Error function. (02)
- b. Prove that $Z(\cos n\theta) = \frac{z(z - \cos \theta)}{z^2 - 2z \cos \theta + 1}$ and $Z(\sin n\theta) = \frac{z \sin n\theta}{z^2 - 2z \cos \theta + 1}$; where $|z| > 1$. (06)
- c. Using laplace transform solve : $y'' + 3y' + 2y = te^{-t}$; where $y(0)=1$ and $y'(0)=0$. (06)

